Second Semester M.Sc. Degree Examination, June 2016
(CBCS)
MATHEMATICS
M 204T : Partial Differential Equations

Time : 3 Hours Max. Marks : 70

Instructions : 1) Answer **any five** questions.
2) **All** questions carry **equal** marks.

1. a) Eliminate c and form the partial differential equation for $(x-c)^2+y^2+z^2=c^2$. 6

b) Solve $yu_x+xu_y=u$ with condition $u(x, 0)=x^3$ 8

2. a) Explain linear, semi linear and quasi linear equations with an example each. 6

b) Find the solution of $u_t+3tu_x=u$, $-\infty<x<\infty$, $t\geq 0$

$u(x)=\begin{cases}\sin x, & 0\leq x\leq \pi\\ 0, & \text{otherwise}\end{cases}$ using the method of characteristics. 8

3. a) Classify the following equations :

i) $\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$ and

ii) $\dfrac{\partial^2 u}{\partial x^2}=x\dfrac{\partial^2 u}{\partial y^2}$ into parabolic or hyperbolic or elliptic equations. 6

b) Reduce $y^2\dfrac{\partial^2 u}{\partial x^2}-2xy\dfrac{\partial^2 u}{\partial x\partial y}+x^2\dfrac{\partial^2 u}{\partial y^2}=\dfrac{y^2}{x}\dfrac{\partial u}{\partial x}+\dfrac{x^2}{y}\dfrac{\partial u}{\partial y}$ to its canonical form. 8

4. a) Using Monge's method, solve $r+s-2t=0$. 7

b) Solve $\dfrac{\partial^3 z}{\partial x^3}-2\dfrac{\partial^3 z}{\partial x^2\partial y}-\dfrac{\partial^3 z}{\partial x\partial y^2}+2\dfrac{\partial^3 z}{\partial y^3}=e^{x+y}$. 7

P.T.O.

5. a) Obtain the solution of Cauchy problem :

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, -\infty < x < \infty, -\infty < t < \infty$$

Subject to

$u(x, 0) = f(x)$

$\frac{\partial u}{\partial t}(x, 0) = g(x), -\infty < x < \infty$ using infinite Fourier transform. 7

b) Solve by variable separable method the IBVP

$$\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}; 0 < x < 1; t > 0$$

Subject to

$$\left.\begin{array}{l} u(x, 0) = x \\ \frac{\partial u}{\partial t}(x, 0) = g(x) \end{array}\right\}; 0 \leq x \leq a,$$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(a, t) = 0 \end{array}\right\}; t \in R.$$ 7

6. a) Obtain the solution of Dirichlet problem $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; -\infty < x < \infty, 0 < y < \infty$ subject to $u(x, 0) = f(x)$; $u \to 0$ as $y \to \infty$ in a half-plane by infinite Fourier transform method. 7

b) Find variable separable solution of the Laplace equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial u}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2 u}{\partial \phi^2}\right) = 0$$ 7

7. a) Make use of appropriate Fourier transform to solve the following IBVP

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}; 0 \leq x < \infty, t \geq 0$$

Subject to

$u(x, 0) = f(x); \ 0 \leq x < \infty$

$\frac{\partial u}{\partial x}(0, t) = 0; t \geq 0$ 7

b) Solve by Fourier decomposition method the following IBVP

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2};\ 0 \le x \le L,\ t \ge 0$$

Subject to

$$u(x, 0) = f(x);\ 0 \le x \le L$$

$$\left.\begin{array}{l} u(0, t) = 0 \\ u(L, t) = 0 \end{array}\right\};\ t \ge 0$$ 7

8. a) Find the Green's function for the following $u_{tt} - 9u_{xx} = Q_1(x);\ -\infty < x < \infty$, $t \ge 0$

Subject to

$$\left.\begin{array}{l} u(x, 0) = 0 \\ u_t(x, 0) = 0 \end{array}\right\};\ -\infty < x < \infty$$

$$\left.\begin{array}{l} u \to 0 \\ \dfrac{\partial u}{\partial x} \to 0 \end{array}\right\} \text{ as } |x| \to \infty,\ t \ge 0.$$ 7

b) Show by using Green's function method that $G(x, t, \xi) = \dfrac{e^{\left(\frac{x-\xi}{\sqrt{4k\pi t}}\right)^2}}{\sqrt{4k\pi t}}$ is the solution of $\dfrac{\partial u}{\partial t} - k\dfrac{\partial^2 u}{\partial x^2} = f(x)\,\delta(t);\ -\infty < x < \infty,\ t > 0$ subject to $u(x, 0) = 0,\ -\infty < x < \infty$. 7
